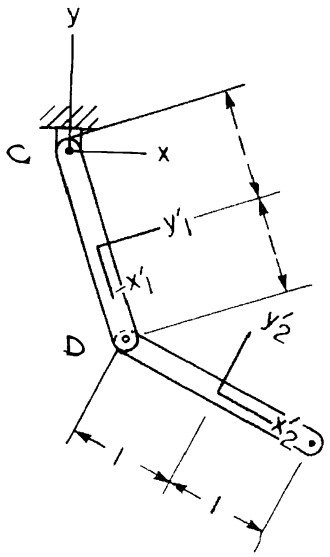
Problem 6.3.3.



$$q = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \\ x_2 \\ y_2 \\ \phi_2 \end{bmatrix}$$

$$J_1' = \frac{m_1 \cdot l^2}{3} = \frac{m_1}{3}$$

$$J_2' = \frac{m_2}{3}.$$

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m_1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{m_2}{3} \end{bmatrix}$$

$$\phi(q_1, q_2) = \begin{bmatrix} x_1 - c\phi_1 \\ y_1 - s\phi_1 \\ x_1 + c\phi_1 - x_2 + c\phi_2 \\ y_1 + s\phi_1 - y_2 + s\phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\phi} = \begin{bmatrix} \dot{x}_1 + \dot{\phi}_1 s\phi_1 \\ \dot{y}_1 - \dot{\phi}_1 c\phi_1 \\ \dot{x}_1 - \dot{\phi}_1 s\phi_1 - \dot{x}_2 - \dot{\phi}_2 s\phi_2 \\ \dot{y}_1 + \dot{\phi}_1 c\phi_1 - \dot{y}_2 + \dot{\phi}_2 c\phi_2 . \end{bmatrix}$$

$$\ddot{\phi} = \begin{bmatrix} \ddot{x}_1 + \ddot{\phi}_1 s\phi_1 + \dot{\phi}_1^2 c\phi_1 \\ \ddot{y}_1 - \ddot{\phi}_1 c\phi_1 + \dot{\phi}_1^2 s\phi_1 \\ \ddot{x}_1 - \ddot{\phi}_1 s\phi_1 - \dot{\phi}_1^2 c\phi_1 - \ddot{x}_2 - \ddot{\phi}_2 s\phi_2 - \dot{\phi}_2^2 c\phi_2 \\ \ddot{y}_1 + \ddot{\phi}_1 c\phi_1 - \dot{\phi}_1^2 s\phi_1 - \ddot{y}_2 + \ddot{\phi}_2 c\phi_2 - \dot{\phi}_2^2 s\phi_2 \end{bmatrix} \Rightarrow \gamma = \begin{bmatrix} -\dot{\phi}_1^2 c\phi_1 \\ -\dot{\phi}_1^2 s\phi_1 \\ \dot{\phi}_1^2 c\phi_1 + \dot{\phi}_2^2 s\phi_2 \\ \dot{\phi}_1^2 s\phi_1 + \dot{\phi}_2^2 s\phi_2 \end{bmatrix}$$

The only force acting on the mechanism is gravity.

Then,

$$Q^A = \begin{bmatrix} 0 \\ -m_1 g \\ 0 \\ 0 \\ -m_2 g \\ 0 \end{bmatrix}$$

The DAE of motion are: $\begin{bmatrix} M & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q^A \\ \gamma \end{bmatrix}$

In other words:

$$\left[\begin{array}{cccccc|cccc}
m_1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & m_1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & \frac{m_1}{3} & 0 & 0 & 0 & s\phi_1 & -c\phi_1 & -s\phi_1 & c\phi_1 \\
0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & \frac{m_2}{3} & 0 & 0 & -s\phi_2 & c\phi_2 \\
\hline
1 & 0 & s\phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -c\phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -s\phi_1 & -1 & 0 & -s\phi_2 & 0 & 0 & 0 & 0 \\
0 & 1 & c\phi_1 & 0 & -1 & c\phi_2 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\phi}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\phi}_2 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ -m_1 g \\ 0 \\ 0 \\ -m_2 g \\ 0 \\ -\dot{\phi}_1^2 c\phi_1 \\ -\dot{\phi}_1^2 s\phi_1 \\ \dot{\phi}_1^2 c\phi_1 + \dot{\phi}_2^2 c\phi_2 \\ \dot{\phi}_1^2 s\phi_1 + \dot{\phi}_2^2 s\phi_2 \end{bmatrix}$$