Example 3.1.1

$$s'^{O} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$r^{O} = r_1 + A s'^{O} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} c\phi_1 & -s\phi_1 \\ s\phi_1 & c\phi_1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 - s\phi_1 \\ y_1 + c\phi_1 \end{bmatrix}$$

$$\Phi(q,t) = \begin{bmatrix} x_1 - s\phi_1 \\ y_1 + c\phi_1 \\ \phi_1 - 4t^2 \end{bmatrix}$$

→ kinematic constraints

→ Driving constraint

x_1, y_1, ϕ_1 → Cartesian generalized coordinates: $q = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \end{bmatrix}$

Velocity Problem:

$$\Phi_q \cdot \dot{q} = \nu$$

$$\Phi_q = \frac{\partial \Phi}{\partial q} = \begin{bmatrix} 1 & 0 & -c\phi_1 \\ 0 & 1 & -s\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \nu = -\Phi_t = \begin{bmatrix} 0 \\ 0 \\ 8t \end{bmatrix}$$

Then, the velocity is solution of

$$\begin{bmatrix} 1 & 0 & -c\phi_1 \\ 0 & 1 & -s\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 8t \end{bmatrix}$$

Acceleration problem:

$$\Phi_q \cdot \ddot{q} = \gamma \quad , \quad \gamma = -(\Phi_q \dot{q})_q \cdot \dot{q} - 2\Phi_{qt}\,\dot{q} - \Phi_{tt}$$

$$(\Phi_q \dot{q})_q = \left(\begin{bmatrix} 1 & 0 & -c\phi_1 \\ 0 & 1 & -s\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \end{bmatrix} \right)_q = \begin{bmatrix} \dot{x}_1 - \dot{\phi}_1 c\phi_1 \\ \dot{y}_1 - \dot{\phi}_1 s\phi_1 \\ \dot{\phi}_1 \end{bmatrix}_q$$

$$= \begin{bmatrix} 0 & 0 & \dot{\phi}_1 s\phi_1 \\ 0 & 0 & -\dot{\phi}_1 c\phi_1 \\ 0 & 0 & 0 \end{bmatrix}$$

Then,

$$(\Phi_q \dot{q})_q \cdot \dot{q} = \begin{bmatrix} 0 & 0 & \dot{\phi}_1 s\phi_1 \\ 0 & 0 & -\dot{\phi}_1 c\phi_1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1^2 s\phi_1 \\ -\dot{\phi}_1^2 c\phi_1 \\ 0 \end{bmatrix}$$

$$\Phi_{qt} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Phi_{tt} = \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}$$

Then, the acceleration $\ddot{q}$ is the solution of

$$\Phi_q \cdot \ddot{q} = \gamma \quad \Rightarrow \quad \begin{bmatrix} 1 & 0 & -c\phi_1 \\ 0 & 1 & -s\phi_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\phi}_1 \end{bmatrix} = \begin{bmatrix} -\dot{\phi}_1^2 s\phi_1 \\ \dot{\phi}_1^2 c\phi_1 \\ 8 \end{bmatrix}$$