Mechanism lock-up example

$$q = \begin{bmatrix} x_2 \\ \phi_1 \\ \phi_2 \end{bmatrix} \qquad \boxed{l = \tfrac{1}{2}.}$$

Constraints → Kinematic: revolute joint at P_1 & P_2.

Constraints → Driving: $\phi_1 = \frac{\pi}{12} t$

$$r_1^P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c\phi_1 & -s\phi_1 \\ s\phi_1 & c\phi_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c\phi_1 \\ s\phi_1 \end{bmatrix}$$

$$r_2^P = \begin{bmatrix} x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} c\phi_2 & -s\phi_2 \\ s\phi_2 & c\phi_2 \end{bmatrix} \begin{bmatrix} 0 \\ l \end{bmatrix} = \begin{bmatrix} x_2 - l\,s\phi_2 \\ l \cdot c\phi_2 \end{bmatrix}$$

Then,

$$\phi(q,t) = \begin{bmatrix} -x_2 + c\phi_1 + l\,s\phi_2 \\ s\phi_1 - l\,c\phi_2 \\ \phi_1 - \frac{\pi}{12} t \end{bmatrix} = 0$$

Constraint Jacobian:

$$\phi_q(q) = \begin{bmatrix} -1 & -s\phi_1 & l\,c\phi_2 \\ 0 & c\phi_1 & l\,s\phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

Then $\det(\phi_q(q)) = l\,s\phi_2 = \frac{1}{2}\,s\phi_2$

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when can this be zero?

$$c\, s\phi_2 = 0 \quad \Rightarrow \quad \phi_2 = 0, \pi, 2\pi, \text{etc.}$$

what happens when $t=2$? we have $\phi_1 = \frac{\pi}{6}$

$$\Rightarrow \quad c\phi_2 = \frac{s\phi_1}{l} = \frac{0.5}{0.5} = 1 \quad \Rightarrow \boxed{\phi_2 = 0.} \quad \Rightarrow \det(\phi_p) = 0$$

$\Rightarrow$ singular configuration.

Let's look at the rank of the augmented Jacobian:

$$J_{vel} = [\,\phi_q \quad -\phi_t\,] = \begin{bmatrix} -1 & -0.5 & 0.5 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{\pi}{12} \end{bmatrix}$$

$$\left.\begin{array}{l} \text{rank}(\phi_p) = 2 \\ \text{rank}(J_{vel}) = 3 \end{array}\right\} \Rightarrow \text{lock-up configuration.}$$

carrying out velocity analysis:

$$\nu = \begin{bmatrix} 0 \\ 0 \\ \frac{\pi}{12} \end{bmatrix}$$

carrying out acceleration analysis:

$$\hat{\phi}(p,t) = \begin{bmatrix} -\hat{x}_2 - \hat{\phi}_1 s\phi_1 + l\,\hat{\phi}_2 c\phi_2 \\ \hat{\phi}_1 c\phi_1 + l\,\hat{\phi}_2 s\phi_2 \\ \hat{\phi}_1 - \frac{\pi}{12} \end{bmatrix}$$

$$\ddot{\Phi}(q,t) = \begin{bmatrix} -\ddot{x}_2 - \ddot{\phi}_1 s\phi_1 - \dot{\phi}_1^2 c\phi_1 + l\ddot{\phi}_2 c\phi_2 - l\dot{\phi}_2^2 s\phi_2 \\ \ddot{\phi}_1 c\phi_1 - \dot{\phi}_1^2 s\phi_1 + l\ddot{\phi}_2 s\phi_2 + l\dot{\phi}_2^2 c\phi_2 \\ \ddot{\phi}_1 \end{bmatrix}$$

Then,

$$\gamma = \begin{bmatrix} \dot{\phi}_1^2 c\phi_1 + l\dot{\phi}_2^2 s\phi_2 \\ \dot{\phi}_1^2 s\phi_1 - l\dot{\phi}_2^2 c\phi_2 \\ 0 \end{bmatrix}$$