Mechanism bifurcation example

Same situation as for mechanism lock-up, except that now the length of the connecting arm is $l=1$. (see "Mechanism Lock-Up Example").

$$\Phi(q,t) = \begin{bmatrix} -x_2 + c\phi_1 + s\phi_2 \\ s\phi_1 - c\phi_2 \\ \phi_1 - \frac{\pi}{12} t \end{bmatrix} = 0$$

$$\Phi_q = \begin{bmatrix} -1 & -s\phi_1 & c\phi_2 \\ 0 & c\phi_1 & s\phi_2 \\ 0 & 1 & 0 \end{bmatrix}.$$

when $t=6 \Rightarrow \phi_1 = \frac{\pi}{2} \Rightarrow \phi_2 = 0 \Rightarrow x_2 = 0$

Then

$$\Phi_q = \begin{bmatrix} -1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \text{Rank } \Phi_q = 2.$$

$$\text{Rank } \hat{J}_{vel} = \text{Rank} \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\pi}{12} \end{bmatrix} = 2.$$

$$\dot{\Phi}(q,t) = \begin{bmatrix} -\dot{x}_2 - \dot{\phi}_1 s\phi_1 + \dot{\phi}_2 c\phi_2 \\ \dot{\phi}_1 c\phi_1 + \dot{\phi}_2 s\phi_2 \\ \dot{\phi}_1 - \frac{\pi}{12} \end{bmatrix}$$

$$\ddot{\phi}_1(q,t) = \begin{bmatrix} -\ddot{x}_2 - \ddot{\phi}_1 s\phi_1 - \dot{\phi}_1^2 c\phi_1 + \ddot{\phi}_2 c\phi_2 - \dot{\phi}_2^2 s\phi_2 \\ \ddot{\phi}_1 c\phi_1 - \dot{\phi}_1^2 s\phi_1 + \ddot{\phi}_2 s\phi_2 + \dot{\phi}_2^2 c\phi_2 \\ \ddot{\phi}_1 \end{bmatrix}$$

Then

$$\gamma = \begin{bmatrix} \dot{\phi}_1^2 c\phi_1 + \dot{\phi}_2^2 s\phi_2 \\ \dot{\phi}_1^2 s\phi_1 - \dot{\phi}_2^2 c\phi_2 \\ 0 \end{bmatrix}$$

For $q = \begin{bmatrix} 0 \\ \pi/2 \\ 0 \end{bmatrix} \Rightarrow \gamma = \begin{bmatrix} \dot{\phi}_1^2 + \dot{\phi}_2^2 \\ 0 \\ 0 \end{bmatrix}$

$$\operatorname{rank} \hat{J}_{acc} = \operatorname{rank} \begin{bmatrix} -1 & -1 & -1 & \dot{\phi}_1^2 + \dot{\phi}_2^2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = 2$$

$\Rightarrow$ $\operatorname{Rank} \hat{J}_{acc} = \operatorname{Rank} \Phi_q = 2$

This situation thus becomes a bifurcation situation, as intuitively anticipated.