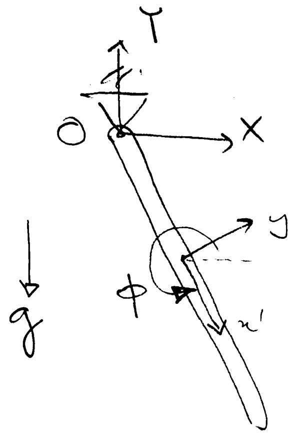
Example 6.3.6.

~ Simple Pendulum ICs ~



To establish a set of consistent IC one has only to consider the constraints associated with the model.

For our model we have a revolute joint between pendulum and ground at point O.

$$r^O = r + A\begin{bmatrix}-l\\0\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}c\phi & -s\phi\\ s\phi & c\phi\end{bmatrix}\begin{bmatrix}-l\\0\end{bmatrix}$$

$$= \begin{bmatrix}x - l\,c\phi\\ y - l\,s\phi\end{bmatrix}$$

The constraints read:

$$\Phi(q) = \begin{bmatrix}x - l\,c\phi\\ y - l\,s\phi\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

At time $t=0$ I want the mechanism to start from a vertical configuration hanging down. I also want it to have an initial angular velocity of $\omega = \dot{\phi} = 2\pi$. These conditions translate at time $t=0$

$$\begin{cases}\phi - \frac{3\pi}{2} = 0\\ \dot{\phi} - 2\pi = 0\end{cases}$$

The position ICs are going then to be the solution of the following set of nonlinear equations:

$$\begin{cases}x_0 - l\,c\phi_0 = 0\\ y - l\,s\phi_0 = 0\\ \phi_0 - \frac{3\pi}{2} = 0\end{cases} \Rightarrow \boxed{\begin{matrix}x_0 = 0\\ y_0 = l\\ \phi_0 = \frac{3\pi}{2}\end{matrix}}$$

As far as the velocity is concerned,

$$\dot{\Phi} = \begin{bmatrix} \dot{x} + \dot{\phi}\, l\, s\phi \\ \dot{y} - \dot{\phi}\, l\, c\phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\dot{\phi}^{IC} = \dot{\phi} - 2\pi = 0$$

Then $\dot{x}, \dot{y}, \dot{\phi}$ is the solution of the following linear system of equations:

$$\begin{cases} \dot{x}_0 + \dot{\phi}_0\, l\, s\phi_0 = 0 \\ \dot{y}_0 - \dot{\phi}_0\, l\, c\phi_0 = 0 \\ \dot{\phi}_0 - 2\pi = 0 \end{cases}$$

Since $\phi_0 = \frac{3\pi}{2}$ $\Rightarrow$

$$\boxed{\begin{aligned} \dot{x}_0 &= l\,\dot{\phi}_0 \\ \dot{y}_0 &= 0 \\ \dot{\phi}_0 &= 2\pi. \end{aligned}}$$