Example 3.5.5

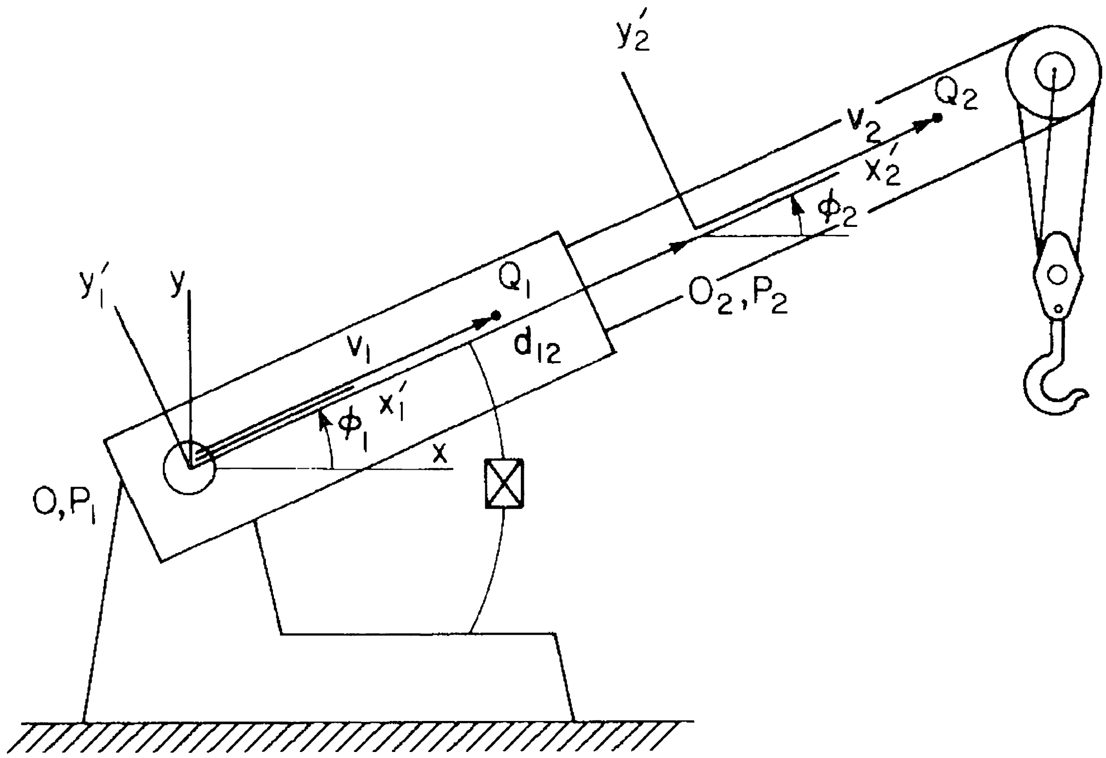

Figure 3.5.10 Wrecker boom with a translational-distance driver.

Generalized coordinates: $q = \begin{bmatrix} \phi_1 \\ x_2 \\ y_2 \\ \phi_2 \end{bmatrix}$.

Constraints, in plain words:

1. The orientation of body 2 is like that of body 1
2. $v_1^\perp$ should stay perpendicular to $\overrightarrow{P_1P_2}$
3. The distance between P_1 & P_2 is given as a function of time.
4. The orientation of the boom changes according to a given function of time.

Constraints, in math language:

1. $\phi_1 - \phi_2 = 0$
2. $(v_1^\perp)^T \cdot (r_1^P - r_2^P) = 0$

3 $\quad v_1^T \cdot (r_2^P - r_1^P) - c(t) = 0$

4 $\quad \phi_1 - 0.025\, t = 0.$

$$v_1 = A_1 v_1' \qquad r_1^P = r_1 + A_1 s_1'^P \qquad r_2^P = r_2 + A_2 s_2'^P$$

$$s_1'^P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad r_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad s_2'^P = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \qquad v_1' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$r_1^P - r_2^P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ -y_2 \end{bmatrix}$$

$$v_1^\perp = R A_1 v_1' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} c\phi_1 & -s\phi_1 \\ s\phi_1 & c\phi_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} c\phi_1 \\ s\phi_1 \end{bmatrix}}_{v_1} = \begin{bmatrix} -s\phi_1 \\ c\phi_1 \end{bmatrix}$$

Then, 2 becomes

$$\begin{bmatrix} -s\phi_1 & c\phi_1 \end{bmatrix} \begin{bmatrix} -x_2 \\ -y_2 \end{bmatrix} = x_2 s\phi_1 - y_2 c\phi_1 = 0.$$

3 becomes

$$\begin{bmatrix} c\phi_1 & s\phi_1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} - c(t) = x_2 c\phi_1 + y_2 s\phi_1 - c(t) = 0.$$

Then the constraints look like:

$$\phi(q,t) = \begin{bmatrix} \phi_1 - \phi_2 \\ x_2 s\phi_1 - y_2 c\phi_1 \\ x_2 c\phi_1 + y_2 s\phi_1 - c(t) \\ \phi_1 - 0.025\, t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Need to derive the velocity equation:

$$\Phi_q \cdot \dot{q} = -\Phi_t .$$

$$\Phi_q = \begin{bmatrix} 1 & 0 & 0 & -1 \\ x_2 \, c\phi_1 + y_2 \, s\phi_1 & s\phi_1 & -c\phi_1 & 0 \\ -x_2 \, s\phi_1 + y_2 \, c\phi_1 & c\phi_1 & s\phi_1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\nu = -\Phi_t = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0.025 \end{bmatrix} .$$

$\det |\Phi_q| = 1 \neq 0 \Rightarrow$ the system $\Phi_q \cdot \dot{q} = \nu$ always has a solution.