Example 3.9.3

When $0 \leq \alpha_1 \leq \frac{2\pi}{3} \Rightarrow \rho_1(\alpha_1) = -\frac{1}{4}\cos 3\alpha_1 + \frac{5}{4}$

Then, $a_1' = \rho_1(\alpha_1) \cdot \begin{bmatrix} c\alpha_1 \\ s\alpha_1 \end{bmatrix} = \rho_1(\alpha_1)\, u_1(\alpha_1) \equiv \rho_1 u_1$

Therefore, $g_1' \equiv \frac{da_1'}{d\alpha_1} = \rho_1' u_1 + \rho_1 u_1^{\perp}$

Likewise,

$$a_2' = \rho_2(\alpha_2)\, u_2(\alpha) = \frac{1}{4} u_2(\alpha) \equiv \frac{1}{4} u_2$$

$$g_2' = \frac{1}{4} \cdot u_2^{\perp}$$

Perpendicularity constraint: $(g_i^{\perp})^T \cdot g_j = 0$

$$g_i = A_i \cdot g_i' \Rightarrow g_i^{\perp} = R A_i g_i'$$

Then,

$$(g_i^{\perp})^T g_j = g_i'^T A_i^T R^T A_j g_j' = -g_i'^T A_i^T R A_j g_j' = -g_i'^T A_i^T A_j R g_j'$$

$$= -g_i'^T A_{ij} R g_j' = -g_i'^T B_{ij} g_j'$$

Then, given the expression of g_i' and g_j', the above constraint assumes the expression

$$-\left[\rho_1' u_1^T + \rho_1 (u_1^{\perp})^T\right] B_{ij} \cdot \rho_2 u_2^{\perp} = 0$$

or,

$$-\left[\rho_1' c\alpha_1 - \rho_1 s\alpha_1 \quad \rho_1' s\alpha_1 + \rho_1 c\alpha_1\right] \begin{bmatrix} -s(\phi_2-\phi_1) & -c(\phi_2-\phi_1) \\ c(\phi_2-\phi_1) & -s(\phi_2-\phi_1) \end{bmatrix} \begin{bmatrix} -\frac{1}{4} s\alpha_2 \\ \frac{1}{4} c\alpha_2 \end{bmatrix} = 0.$$

$\rho_1' = \frac{3}{4} \sin 3\alpha_1$ $\qquad \rho_1 = -\frac{1}{4}\cos 3\alpha_1 + \frac{5}{4}$