EXAM 2: ME451 – SPRING 2009
MAX SCORE: 60 POINTS

Problem 1. Provide short answers to the following questions (3 points for each correct answer for a total of 15 points):

1. Consider the following initial value problem (IVP): $\begin{cases} \dot{y} + 100y = 0 \\ y(0) = 1 \end{cases}$

Suppose that you use an integration step size $\Delta t = 0.01$ seconds to find an approximation of the solution of this IVP. What would be the value of the approximate solution produced after *one* integration step; i.e., at $t = 0.01$, by each of the following two numerical integration methods:

a) Forward Euler

b) Backward Euler

2. Given the initial value problem above, which of the two numerical integration schemes would allow one to advance in a stable fashion the numerical solution with a larger step-size and why?

3. Recall that we had to qualify the nature of the virtual displacements that we worked with in the virtual work approach to get rid of the reaction forces. What did we have to do that, and why did that eliminate the constraint reaction forces from the picture?

4. Do you always have to express the equations of motion with respect to the Center of Mass associated with each of the bodies present in the system? Explain your answer.

5. In your own words, what is the Lagrange Multiplier Theorem saying? Why is this theorem relevant in the context of deriving the equations of motion?

Problem 2 (20 points)

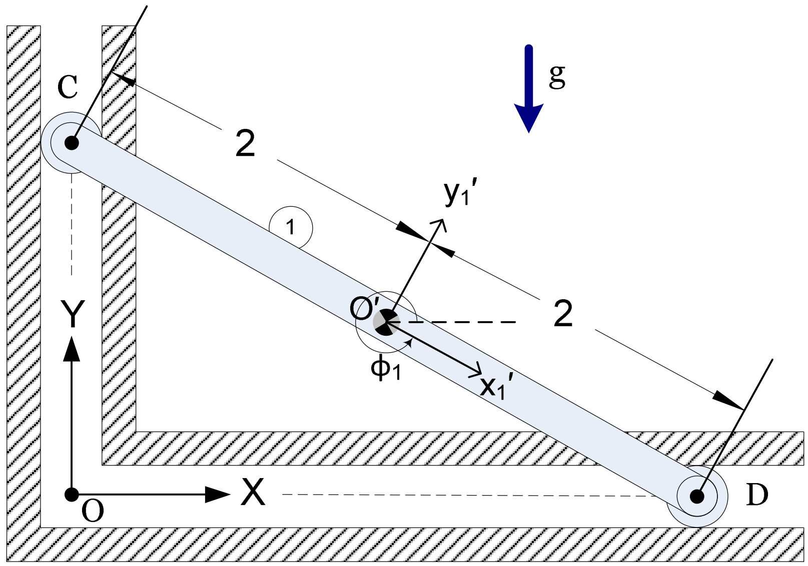

Figure 1. One body mechanism, torque applied. Front view.

For the mechanism in **Figure 1**, ignore the rollers at point **C** and **D** whose role is to insure that the body in the figure moves along the grooves with no friction. A motion is applied to this body so that it leads to a time evolution of the angle like $\phi_1(t) = \frac{\pi}{15}t + \frac{5\pi}{3}$. The lengths are as provided in the figure. The center of mass (CM) is located in the middle of the symmetric link, as shown in the figure. For this problem, consider the following set of generalized coordinates: $\mathbf{q} = [x_1, y_1, \phi_1]^T$, where x_1, y_1 are the coordinates of the CM of body 1 in the global fixed reference frame OXY, and the angle ϕ_1 provides the orientation of the body with respect to the same global fixed reference frame. At time $t = 0$ the initial velocity of the mechanism is 0, while the orientation angle is $\phi_1 = \frac{5\pi}{3}$. The mass of the body is 1, the mass moment of inertia about the CM is 1, and the gravitational acceleration is $g = 9.8$. All units are SI.

a) Given your set of generalized coordinates, specify the set of constraints $\Phi(\mathbf{q}, t) = \mathbf{0}$ associated with the kinematics of the mechanism.

b) Derive the acceleration kinematic constraint equations

c) Derive the equations of motion for this mechanism

d) How do you compute the torque $n(t)$ required to enforce the motion specified (i.e., $\phi_1 = \frac{\pi}{15}t + \frac{5\pi}{3}$)?

Problem 3 (25 points)

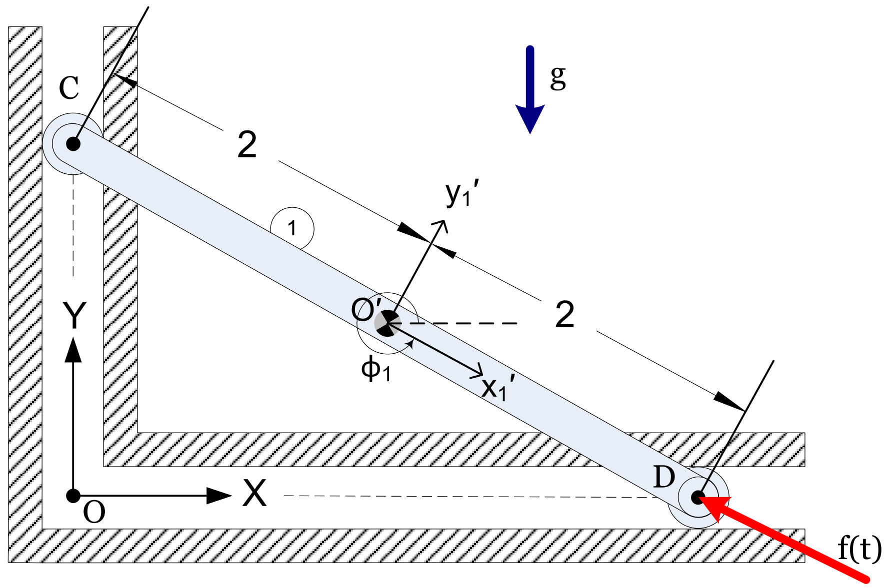

Figure 2. One body mechanism, force applied. Front view.

For the same mechanism as in the previous problem, consider now the case when there is no motion specified (no explicit motion governs the evolution of ϕ_1). Instead, there is a force that is *always* applied along the line from D to C (see figure). The magnitude of this force is constant: $f(t) = 4$. All units, like in the previous problem, are SI.

a) Given your set of generalized coordinates, specify the set of constraints $\Phi(\mathbf{q},t) = \mathbf{0}$ associated with the kinematics of the mechanism
b) Formulate the equations of motion for this problem
c) What is the reaction force at time $t = 0$ at point C? Consider the same initial conditions provided in the previous problem.
d) Compute the value of the acceleration $\ddot{\mathbf{q}}$ at time $t = 0$