

# Equilibrium Example.

01/2

Generalized coordinates:

$$q = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix}$$

Constraints:

- revolute joint at origin

$$\Phi(q) = \begin{bmatrix} x - L \cos \phi \\ y - L \sin \phi \end{bmatrix} = \mathbf{0}_{2 \times 1}$$

Generalized forces acting on the pendulum:

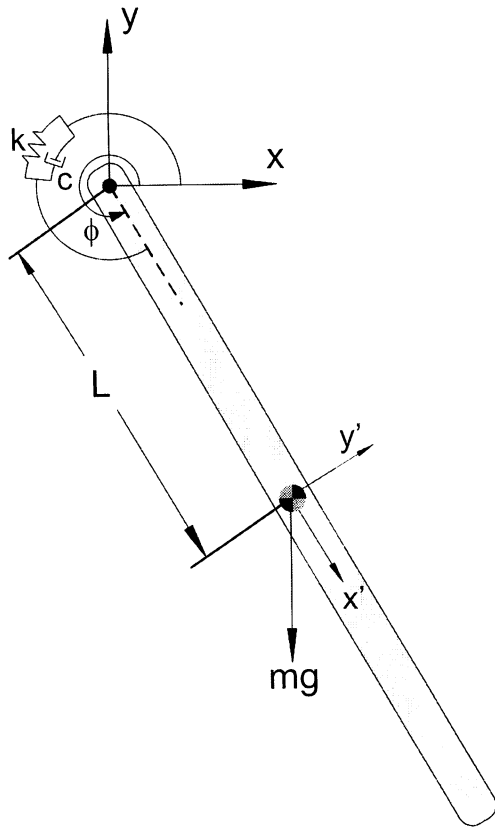
$$Q^A = \begin{bmatrix} 0 \\ -mg \\ -k(\phi - \phi_{free}) - c\dot{\phi} \end{bmatrix}$$

$$m = 1$$

$$g = 9.81$$

$$k = 25$$

$$L = 1$$



At equilibrium,

$$\begin{cases} \Phi_p^T \lambda = Q^A \\ \Phi(q) = 0 \end{cases}$$

$$\Phi_p = \begin{bmatrix} 1 & 0 & L \sin \phi \\ 0 & 1 & -L \cos \phi \end{bmatrix}$$

$$\Rightarrow \Phi_p^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ L \sin \phi & -L \cos \phi \end{bmatrix}$$

We have 5 unknowns:  $x, y, \phi, \lambda_1, \lambda_2$

We have 5 equations (nonlinear).

$$\begin{cases} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ L \sin \phi & -L \cos \phi \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -mg \\ -k(\phi - \phi_{free}) - c\dot{\phi} \end{bmatrix} \end{cases}$$

$$\begin{cases} x - L \cos \phi = 0 \\ y - L \sin \phi = 0 \end{cases}$$

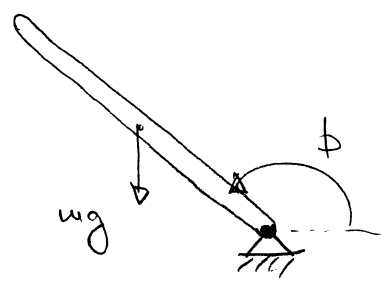
$$\Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = -\omega g \\ L \cancel{\lambda_1} \phi - L \lambda_2 c\phi = -k(\phi - \phi_{free}) \\ x - L c\phi = 0 \\ y - L s\phi = 0 \end{cases}$$

Then,  $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\omega g \end{bmatrix}$

$$-L(-\omega g) c\phi = -k\phi \Rightarrow \boxed{\omega g L c\phi = -k\phi}$$

The equilibrium configuration will therefore look like

For the given values, the equation that needs to be solved is



$$98.1 \cdot c\phi + 25 \phi = 0$$

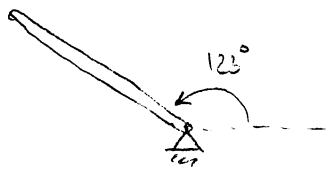
Using MATLAB's "fzero", one gets two solutions:

$$\phi^{(1)} = 2.1507 \text{ [rad]}$$

⇓

$$\phi^{(1)} = 123.24^\circ$$

⇓



$$\phi^{(2)} = 3.5698 \text{ [rad]}$$

⇓

$$\phi^{(2)} = 204.53^\circ$$

⇓

