EXAM 2 – ME451
Take Home Exam Component
Spring 2009

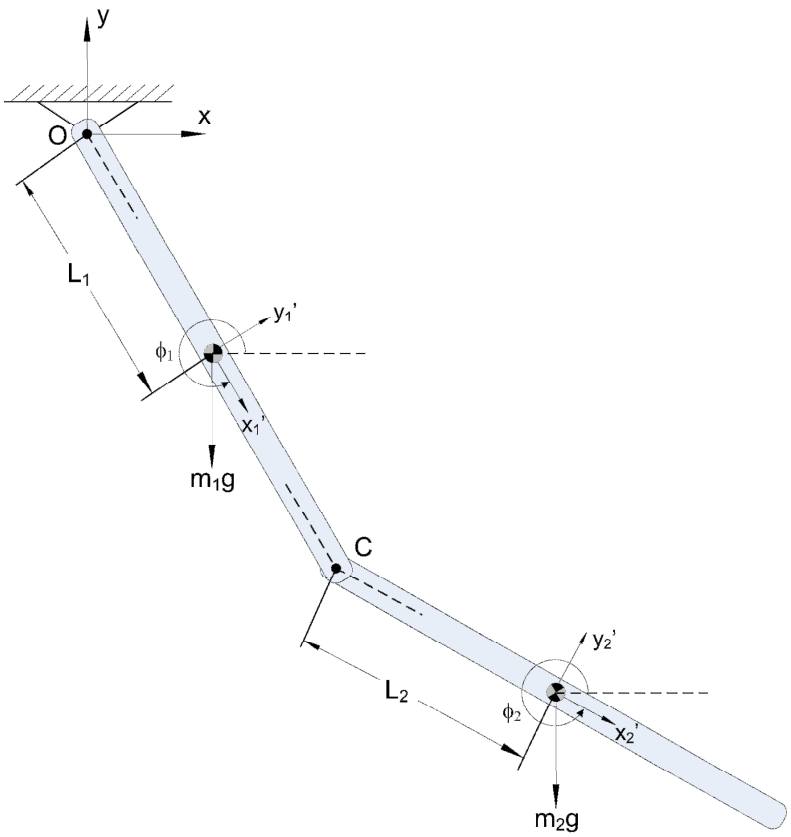

Figure 1. Double Pendulum Mechanism

For the two dimensional double pendulum mechanism in Figure 1, the model parameters are as follows: $L_1 = 0.5$, $m_1 = 1$, $L_2 = 1$, $m_2 = 4$. In the initial configuration the double pendulum hangs down. Pendulum 1 is at a complete rest, while pendulum 2 has an initial translational velocity in the global X-direction of 10. The gravitational acceleration is $g = 9.81$. All units are SI. The assumption is that there is no friction in the joints of the mechanism.

1. Derive the equations that govern the time evolution of this mechanical system (equations of motion, position constraints, velocity constraints, acceleration constraints).
2. Indicate the initial value of each of your generalized coordinates at the beginning of the simulation ($t = 0$).
3. In MATLAB, using Newmark's formula with $\gamma = 0.6$ and $\beta = 0.3025$:

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + \frac{h^2}{2}\left[(1-2\beta)\,\ddot{\mathbf{q}}_n + 2\beta\ddot{\mathbf{q}}_{n+1}\right]$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\left[(1-\gamma)\,\ddot{\mathbf{q}}_n + \gamma\ddot{\mathbf{q}}_{n+1}\right],$$

find the time evolution of the mechanism under gravitational force for 20 seconds.
4. Generate the ADAMS model of the same mechanism and validate results obtained in MATLAB.

What should you turn in?

a) Work that shows how the constrained equations of motion as well as the initial conditions were obtained.
b) Plot showing time evolution of the angles of the two pendulums in MATLAB and ADAMS.
c) Plot showing time evolution of velocity in the X direction of the center of mass of body 2 in MATLAB and ADAMS.
d) Plot showing the number of iterations for convergence in MATALB.
e) Plots showing reaction force and torque in joint C (these components should be expressed in the global reference frame) in MATLAB and ADAMS.

Remarks:

i) For solving this take-home component of Exam 2 you cannot team up with anybody else in the class. The solution should reflect your personal understanding of the subject, as well as your code design choices.
ii) Please bring your work to the Final Exam, and email the MATLAB code (in a zipped file) and ADAMS model to the TA prior to the beginning of the exam on that day.
iii) Use the example posted at Learn@UW ("codeSimplePendulum.zip") and discussed in class to help setting up the Newmark-based MATLAB implementation that solves numerically for the time evolution of the double pendulum. The posted example is challenging due to the three generalized forces acting on the mechanism. This take-home is challenging in that the dimension of the resulting problem is higher. However, this should not be an issue if you use MATLAB to solve the problem.