A vertical suction pipe 3.25 m long with an outside diameter of 0.15 m is submerged to a depth of 1.25 m in a river that is flowing at 3 mph as shown in the accompanying figure. It is attached to a horizontal pipe that has a torsional spring constant of \( k_c = 15,000 \text{ Nm/rad} \). A simplified model of the two pipes is shown in part b of the figure. Vortex shedding subjects the system to a moment \( M_t \) about the \( z \) axis shown due to the force

\[
F = \frac{C p D^2}{2} \sin(2 \pi \Omega t)
\]

where \( C = 1.0 \) and \( \rho = 1000 \text{ kg/m}^3 \).

Determine the steady-state amplitude of vibration \( \theta_0 \) of the suction pipe if the fundamental natural frequency of the system is 3.1 Hz and the damping factor is \( \zeta = 0.25 \).

\[ \text{Ans. } \theta_0 = 0.17 \text{ rad} \]

\[ \text{Also, neglecting } W \text{ in comparison to } k_c: \]

\[ I_o \ddot{\theta} + k_c \theta = F(t) \]

\[ \theta + k_c \theta = \frac{F(t)}{I_o} \sin(2 \pi \Omega t) \]

The response is given by:

\[ \theta(t) = \frac{F(t) \Omega^2}{\sqrt{(1 - \Omega^2)^2 + 4 \zeta^2 \Omega^2}} \]

\[ \Omega = \sqrt{\frac{k}{J}} = \sqrt{\frac{15,000}{1500}} = 3.1 \text{ Hz} \]

\[ \zeta = 0.25 \]

\[ d_1 = 2.0 \times 0.625 = 2.625 \text{ m} \]

\[ d_2 = 2.0 \times 0.625 = 2.0 \text{ m} \]

\[ k_c = 15,000 \text{ Nm/rad} \]

\[ C = 1000 \text{ kg/m}^3 \]

\[ \text{Determine flow parameters:} \]

\[ F_0 = \frac{C p D^2 A}{2} \]

\[ \Omega = 3.1 \text{ Hz} \]

\[ C = 1000 \text{ kg/m}^3 \]

\[ \frac{d_1}{d_2} = \frac{2.625}{2.0} = 1.3125 \]

\[ \frac{1}{3.125} \times \frac{2.24}{1.0} = 2.24 \text{ m/s} \]

\[ A = \text{projected area} = 1.25(0.15) = 0.188 \text{ m}^2 \]

\[ F_0 = \frac{(1)(1000)(2.24)^2}{2} (0.188) = 472 \text{ N} \]

\[ \text{Check the Reynolds number of the fluid:} \]

\[ \Re = \frac{v d_1}{\mu} = \frac{2.24 \text{ m/s}}{1.1 \times 10^{-3} \text{ Ns/m}^2} \]

\[ R_e = 336,000 \Rightarrow S \approx 0.2 \]
- Calculate the excitation frequency -
  \[ f_0 = \frac{S_n}{2\pi \ell} = 0.2 \left( \frac{2.24 \text{ m/sec}}{0.15 \text{ m}} \right) = 2.987 \text{ Hz} \]

- Calculate frequency ratio - \( \nu \)
  \[ \nu = \frac{\omega_n}{\omega_p} = \frac{2.987}{3.1} = 0.9635 \]

- Substituting \( \nu \) back into the eqn for the response amplitude -
  \[ |\theta| = \frac{472(2.425)/18,000}{\sqrt{[1 - (0.9635)^2]^2 + [2(0.25)(0.9635)]^2}} = 0.17 \text{ rad} \]

  The steady-state amplitude is: \[ \theta = 0.17 \text{ rad} \]

  Recalling that \( k_x = \frac{GJ}{I} \) for the horizontal pipe, it is suggested that the length of the horizontal pipe be increased from \( \ell \) to \( 2\ell \) as a means of reducing the amplitude of vibration. Do you agree with this suggestion if the flow velocity has a range of 5 mph to 10 mph?

**Part B - Increasing the length \( \ell \) of the pipe to \( 2\ell \)**

- \( \omega_n = \sqrt{\frac{k_n}{m}} \) where \( k_n = \frac{GJ}{2\pi \ell} \)

- For a length \( \ell = 2\ell \)
  \[ \omega_n = \sqrt{\frac{k_n}{2\pi \ell}} = \frac{\omega_n}{\sqrt{2}} \]

- Then \( \frac{f_n}{f_n'} = \frac{3.1 \text{ Hz}}{\sqrt{2}} = 2.192 \text{ Hz} \)

Recalculate frequency ratio - \( \nu' \) (for 5 mph velocity)

\[ \nu' = \frac{f_n'}{f_n} = \frac{2.987 \text{ Hz}}{2.192 \text{ Hz}} = 1.363 \]

\[ |\theta'| = \frac{472(2.425)/18,000}{\sqrt{[1 - (1.363)^2]^2 + [2(0.25)(1.363)]^2}} = 0.151 \text{ rad} \]

\[ \theta' = 0.151 \text{ rad} \] for \( v = 5 \text{ mph} \)  

Yes, amplitude is reduced.

Check amplitude at 10 mph for \( \nu' \)

\[ v = 10 \text{ mph} = 4.48 \text{ m/sec} \]

\[ F_0'' = \frac{(4)(1000)(4.48)^2(0.186)}{2} = 186.6 \]

\[ \beta'' = \frac{v_1 f'_{n'}}{m} = \frac{4.48 (0.186)}{1 \times 10^{-3}} = 672,000 \]

\[ S'' = 0.24 \]

\[ f'' = \frac{S'' v''}{d} = \frac{0.24 (4.48)}{0.15} = 7.168 \text{ Hz} \]

\[ \nu'' = \frac{\omega''}{\omega_p} = \frac{f''}{f_n} = \frac{7.168 \text{ Hz}}{2.192 \text{ Hz}} = 3.27 \]

\[ |\theta''| = \frac{186.6(2.425)/18,000}{\sqrt{[1 - (3.27)^2]^2 + [2(0.25)(3.27)]^2}} = 0.067 \text{ rad} \]

\[ \theta'' = 0.067 \text{ rad} \] for \( v = 10 \text{ mph} \)  

Yes, amplitude is reduced.