Considering an undamped system,
\[ x(t) = \int_{0}^{t} F(\tau) g(t-\tau) d\tau \]
\[ g(t) = \frac{1}{m \omega_n} \sin \omega t \]
\[ = \frac{F_0}{m \omega_n} \sin \omega t \]
\[ = \frac{F_0}{m \omega_n} \left[ 1 - e^{-\omega_n t} \right] \]

Substituting into:
\[ x(t) = \int_{0}^{t} F(\tau) g(t-\tau) d\tau \]
we have:
\[ x(t) = \frac{F_0}{m \omega_n} \int_{0}^{t} \sin \omega_n (t-\tau) d\tau = \frac{F_0}{m \omega_n} \left[ \frac{\cos \omega_n (t-\tau)}{\omega_n} \right]_{0}^{t} \]
\[ = \frac{F_0}{m \omega_n} \left[ 1 - e^{-\omega_n t} \right] \]

The result indicates that the peak response to the step excitation of magnitude \( F_0 \) is equal to twice the static deflection.

Considering a damped system,
\[ g(t) = \frac{1}{m \omega_n} e^{-\xi \omega_n t} \sin \omega t \]

Substituting into:
\[ x(t) = \int_{0}^{t} F(\tau) g(t-\tau) d\tau \]
we have:
\[ x(t) = \frac{F_0}{m \omega_n} \int_{0}^{t} e^{-\xi \omega_n (t-\tau)} \sin \omega_n (t-\tau) d\tau \]

Set \( t-\tau = 1 \) and \( d\tau = dt \)
\[ x(t) = \frac{F_0}{m \omega_n} \int_{0}^{t} e^{-\xi \omega_n \tau} \sin \omega_n \tau \ d\tau \]

This yields,
\[ x(t) = \frac{F_0}{m \omega_n} \left[ \frac{e^{-\xi \omega_n t} \left( \omega_n - i \omega_n \right) e^{-\xi \omega_n t}}{m \omega_n} - \frac{e^{-\xi \omega_n t} \left( \omega_n + i \omega_n \right) e^{-\xi \omega_n t}}{m \omega_n} \right] \]

Another way to derive the solution to the step function is to consider the differential equation of motion as
\[ \ddot{x} + 2\xi \omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{m} \]

Again,
\[ x(t) = x_p + x_p' \]

By inspection, \( x_p = \frac{F_0}{m \omega_n^2} \)

Then, the complete solution is:
\[ x(t) = e^{-\xi \omega_n t} \left( B \cos \omega_n t + B' \sin \omega_n t \right) + \frac{F_0}{m} \]
To the initial condition $x_0, \dot{x}_0 = 0$ we find

$$\frac{B_t}{k} = -\frac{F_0}{k} \quad \frac{B_s}{k} = -\frac{F_s}{k} \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

Substitution yields:

$$x(t) = \frac{F_0}{k} \left[ 1 - e^{-\zeta \omega_n t} \left( \cos \omega_d t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \omega_d t \right) \right]$$

$\alpha t$

$$x(t) = \frac{F_0}{k} \left[ \frac{\zeta}{\sqrt{1 - \zeta^2}} \left( \cos (\omega_d t - \psi) \right) \right] \quad \text{Same checked.}$$

Where $\psi = \tan^{-1} \frac{\zeta}{\sqrt{1 - \zeta^2}}$

Graphically:

[Graph showing oscillatory behavior with different damping factors]