The response to the unit impulse is of importance to
the problems of transients, and is designated $g(t)$
Thus, in either the damped or undamped case, the
equation for the impulse response can be
expressed in the form
$$x(t) = F g(t)$$
This can be rewritten as
$$x(t) = \int_{-\infty}^{t} F(\tau) g(t-\tau) \, d\tau$$
This is known as the convolution integral or superposition
integrand.

Another form of the equation is found by letting
$$t + \tau = \lambda \quad t - \tau = \lambda$$
Note: when $\lambda = 0$ it is the same.
When $\lambda \neq 0$ the order of integration is reversed
$$x(t) = \int_{-\infty}^{t} F(\lambda - \tau) g(\lambda) \, d\lambda$$
Since $\lambda$ and $t$ are dummy variables of integration, $t$ and
$\lambda$ are equal and can be rewritten as
$$x(t) = \int_{-\infty}^{t} F(\tau) g(t - \tau) \, d\tau = \int_{0}^{\infty} F(\tau) g(t - \tau) \, d\tau$$

To determine the response of a single degree of freedom
system, consider the principle of superposition. The
response of the two functions should be added.

Example 1 - Step Excitation
Determine the response of a single degree of freedom
system to the step function:
$$f(t)$$
$$F(t)$$

...
Considering an undamped system, \( x(t) = \int_0^t F(t) \, g(t) \, dt \)
\[
g(t) = \frac{1}{m \omega_n} \sin \omega_n t
\]
\[
= \frac{1}{m \omega_n} \left[ 1 - \cos \omega_n t \right]
\]
Substituting into: \( x(t) = \frac{\int_0^t F(t) \, g(t) \, dt}{m \omega_n} \)
\[
= \frac{1}{m \omega_n} \int_0^t \sin \omega_n (t - \tau) \, d\tau
\]
\[
= \frac{1}{m \omega_n} \int_0^t \cos \omega_n (t - \tau) \, d\tau
\]
\[
= \frac{1}{m \omega_n} \left[ \frac{1}{1 + \omega_n^2} \right]
\]
The result indicates that the peak response to the input excitation of magnitude \( F_0 \) is equal to twice the static deflection.

Considering a damped system,
\[
g(t) = \frac{1}{m \omega_d} e^{-\zeta \omega_d t} \sin \omega_d t
\]
Substituting into: \( x(t) = \int_0^t F(t) \, g(t + \tau) \, d\tau \)
\[
x(t) = \frac{F_0}{m \omega_d} \int_0^t e^{-\zeta \omega_d (t - \tau)} \sin \omega_d (t - \tau) \, d\tau
\]
This yields,
\[
x(t) = \frac{F_0}{\omega_d} \int_0^t e^{-\zeta \omega_d (t - \tau)} \left[ e^{j \omega_d \tau} - e^{-j \omega_d \tau} \right] \, d\tau
\]
\[
= \frac{F_0}{\omega_d} \int_0^t e^{j \omega_d (\zeta t - \tau)} - e^{-j \omega_d (\zeta t - \tau)} \, d\tau
\]
\[
= \frac{F_0}{\omega_d} \left[ \frac{e^{j \omega_d (\zeta t - t)} - e^{j \omega_d (\zeta t + t)}}{j \omega_d} \right]
\]
where \( \psi = \tan^{-1} \frac{\zeta}{\sqrt{1 - \zeta^2}} \).