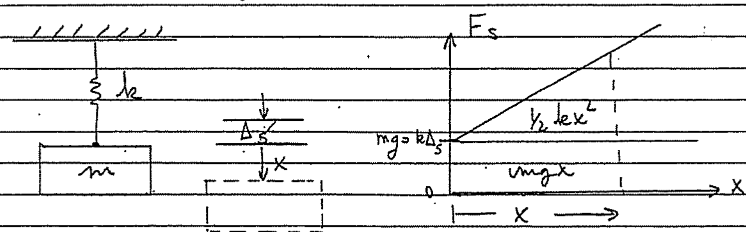
Example 1: Determine the differential equation of motion of the system:



Chose the static equilibrium position to be the reference of zero potential energy. Due to the displacement x from this reference the increase in the potential energy of the spring (area under the curve) is $mgx + \frac{1}{2} kx^2$.

$$U_{sp} = mgx + \frac{1}{2} kx^2$$

But the loss in potential energy due to the weight is

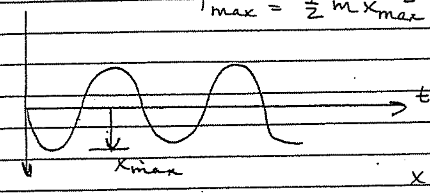
$$U_g = -mgx$$

$$U = mgx + \frac{1}{2} kx^2 - mgx = \frac{1}{2} kx^2.$$

$$U_{max} = \frac{1}{2} k x_{max}^2$$

The kinetic energy is $T = \frac{1}{2} m \dot{x}^2$

$$T_{max} = \frac{1}{2} m \dot{x}_{max}^2$$



$$x = x_{max} \sin \omega_n t$$

$$\dot{x} = x_{max} \omega_n \cos \omega_n t$$

$$T_{max} = \frac{1}{2} m (x_{max} \omega_n)^2$$

Then, $T_{max} = U_{max}$ (to determine ω_n)

$$\frac{1}{2} m (x_{max} \omega_n)^2 = \frac{1}{2} k x_{max}^2$$

$$m \omega_n^2 = k$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}} \Leftarrow$$

To determine the equation of motion:

$$\frac{d}{dt}\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} kx^2\right) = 0$$

$$\boxed{m\ddot{x} + kx = 0} \Leftarrow$$