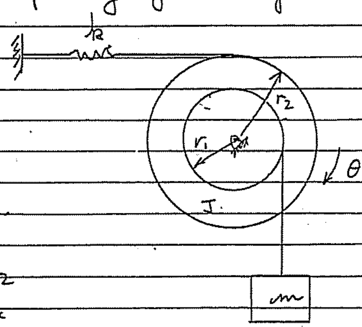
Determine the natural frequency of the system shown:



Assume that the system is vibrating harmonically with amplitude θ from its static equilibrium position.

$$T_{max} = \frac{1}{2} J \dot{\theta}^2_{max} + \frac{1}{2} m (r_1 \dot{\theta})^2_{max}$$

$$\theta = \theta_{max} \sin \omega_n t$$

$$U_{max} = \frac{1}{2} k (r_2 \theta)^2_{max}$$

$$\dot{\theta} = \theta_{max} \omega_n \cos \omega_n t \qquad \dot{\theta}_{max} = \theta_{max} \omega_n$$

$$v = r_1 \dot{\theta}_{max} \sin \omega_n t \qquad v_{max} = r_1 \dot{\theta}_{max}$$

$$T_{max} = U_{max}$$

$$\frac{1}{2} J \dot{\theta}^2_{max} + \frac{1}{2} m (r_1 \dot{\theta})^2_{max} = \frac{1}{2} k (r_2 \theta)^2_{max}$$

$$(J + m r_1^2) \dot{\theta}^2_{max} = k r_2^2 \theta^2_{max}$$

$$(J + m r_1^2)(\cancel{\theta_{max}} \omega_n)^2 = k r_2^2 \cancel{\theta^2_{max}}$$

$$\omega_n^2 = k r_2^2 / (J + m r_1^2)$$

$$\boxed{\omega_n = \sqrt{k r_2^2 / (J + m r_1^2)}}$$