Example 4

A four-bladed aluminum propeller of a wind turbine is supported on a small shaft so that it is free to rotate about its central axis. A weight \( W = 20 \) lb is suspended to one of the blades at a distance \( R = 72 \) in. from the axis of rotation as shown. When the unbalanced weight \( W \) is displaced slightly from the vertical axis shown, the propeller is found to oscillate 10 cycles in 30.5 s. Determine the centroidal mass moment of inertia \( I \) of the propeller.

F.B.D.

Assume small \( \theta \) and neglect damping.

\[ \Sigma M = I \cdot \ddot{\theta} = (\Sigma M)_{eff} \]

\[ -WR \sin \theta \cdot R = I \cdot \ddot{\theta} + \frac{W}{g} R^2 \ddot{\theta} \]

\[ I \cdot \ddot{\theta} + \frac{W}{g} R^2 \ddot{\theta} + WR \sin \theta = 0 \]

but \( \sin \theta \approx \theta \)

\[ \ddot{\theta} \left( I + \frac{W}{g} R^2 \right) + WR \theta = 0 \]

\[ \ddot{\theta} + \frac{WR}{I + \frac{W}{g} R^2} \theta = 0 \]

but

\[ \omega_n^2 = \frac{WR}{I + \frac{W}{g} R^2} \]

\[ \tau_n^2 = \left( \frac{2\pi}{\omega_n} \right)^2 = \left( \frac{2\pi}{\omega} \right)^2 = \frac{(2\pi)^2}{WR} \left( I + \frac{W}{g} R^2 \right) \]

\[ \overline{I} = \frac{\tau_n^2}{(2\pi)^2} \cdot \frac{WR - \frac{WR^2}{g}}{\theta} = 601.6 \text{ lb} \cdot \text{in} \cdot \text{s}^2 \]

Example 5:

The device shown in the accompanying figure is proposed as a means of determining the weight \( W \) and centroidal mass moment of inertia \( I \) of rubber-tired wheels of various sizes by recording the frequency of oscillation of the wheels both when free to rotate about an axle fixed to the horizontal bar \( AB \) and when locked to the bar through the axle. Determine the differential equations of motion for both a free and a locked wheel, and see if it is possible to determine \( W \) and \( I \) for a wheel by measuring the respective frequencies.

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