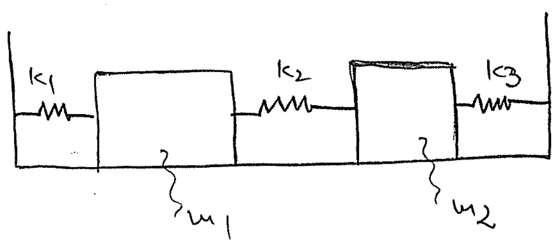
Example

Matrix vector approach to 2DOF Vibration Problem



EOM: (was derived in a previous example)

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$m_1 = 1 \qquad m_2 = 2$

$k_1 = 9 \qquad k_2 = k_3 = 18$

(units: SI)

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \qquad [K] = \begin{bmatrix} 27 & -18 \\ -18 & 36 \end{bmatrix}$$

$$[M^{-1}] = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \Rightarrow [A] = \begin{bmatrix} 27 & -18 \\ -9 & 18 \end{bmatrix}$$

Solve for natural frequencies:

$$\det([A] - \omega_n^2 [I]) = 0 \Rightarrow \det\left(\begin{bmatrix} 27 & -18 \\ -9 & 18 \end{bmatrix} - \begin{bmatrix} \omega_n^2 & 0 \\ 0 & \omega_n^2 \end{bmatrix} \right) = 0$$

$$\Rightarrow \det \begin{bmatrix} 27-\omega_n^2 & -18 \\ -9 & 18-\omega_n^2 \end{bmatrix} = 0$$

$$\Rightarrow (27-\omega_n^2)(18-\omega_n^2) - 162 = 0 \Rightarrow \begin{cases} \omega_{n(1)} = 3 \\ \omega_{n(2)} = 6 \end{cases}$$

The eigenvector for $\omega_{n(1)}^2$ is:

$$[A]\{u\}_{(1)} = \omega_{n(1)}^2 \{u\}_{(1)} \Rightarrow \begin{bmatrix} 27 & -18 \\ -9 & 18 \end{bmatrix} \begin{Bmatrix} 1 \\ r_{(1)} \end{Bmatrix} = 9 \begin{Bmatrix} 1 \\ r_{(1)} \end{Bmatrix}$$

$$\Rightarrow 27 - 18 r_{(1)} = 9 \Rightarrow r_{(1)} = 1. \Rightarrow \{u\}_{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}.$$

similarly,

$$[A]\{u\}_{(2)} = \omega_{n(2)}^2 \{u\}_{(2)} \Rightarrow \begin{bmatrix} 27 & -18 \\ -9 & 18 \end{bmatrix} \begin{Bmatrix} 1 \\ r_{(2)} \end{Bmatrix} = 36 \begin{Bmatrix} 1 \\ r_{(2)} \end{Bmatrix} \Rightarrow \{u\}_{(2)} = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}.$$

Then $[u] = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix}$.

For this system, $[\Omega] = \begin{bmatrix} 9 & 0 \\ 0 & 36 \end{bmatrix}$.

The identity $[A][u] = [u][\Omega]$ becomes

$$\begin{bmatrix} 27 & -18 \\ -9 & 18 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -0.5 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 36 \end{bmatrix},$$

which is easy to see that is true.

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