Consequently, the ratio of the amplitude of the response $x_p(t)$ to that of the base motion $y(t)$ is:

$$\frac{x_p}{y} = \left[ \frac{\sqrt{1 + \left(2\pi f_0\right)^2}}{\left(1 - \rho^2\right)^2 + (2\pi f_0)^2} \right] \left[ \frac{1 + \left(2\pi f_0\right)^2}{\left(1 - \rho^2\right)^2 + (2\pi f_0)^2} \right]^{\frac{1}{2}}$$

**Example Problem 1**

An automobile is modeled as a single degree of freedom system vibrating in the vertical direction. It is driven along a road whose elevation varies sinusoidally. The distance from peak to trough is 0.2 m and the distance along the road between peaks is 35 m. If the natural frequency of the automobile is 2 Hz and the damping ratio of the shock absorbers is 0.15, determine the amplitude of vibration of the automobile at a speed of 60 km/hr. If the speed of the automobile is varied, find the most unfavorable speed for the passengers, i.e., when is the amplitude of motion is the largest.

**Example Problem 2**

An instrument package of mass $m$ is mounted on a support structure of stiffness $k$ in the nose of a rocket as shown in part (a) of the accompanying figure. The support structure, which could involve springs, beams, plates, and so forth, is represented schematically by the spring-and-mass system shown in part b of the figure. Assuming that the nose cone to which the support structure is rigidly fastened experiences the acceleration shown in part c of the figure, and neglecting damping, determine (a) an expression for the relative displacement $z$ of the instrument package with respect to the rocket and (b) the absolute acceleration $\ddot{x}$ of the instrument package as a function of time. If $z$ is to be kept small, should $k/m$ be large or small?

Partial ans: $\ddot{x} = a_0 \left(1 - \frac{1}{\omega_0 f} \sin \omega_0 t\right)$

![Diagram of instrument package](image)
\[ \ddot{x} = m \ddot{v} \]
\[ -k(x-y) + m \ddot{v} = m(\dddot{x} + \dddot{y}) \]
\[ h(x-y) \]
\[ \dddot{x} + \frac{k}{m} x = -\dddot{y} \]

But from the graph, \( \dddot{y} = a_0 t \)

The equation of motion becomes:

\[ \dddot{x} + \frac{k}{m} x = -a_0 t \]

The solution is given by:

\[ \ddot{x} = A \cos \omega_n t + B \sin \omega_n t - \frac{a_0}{\omega_n^2} \left( \frac{\omega_n^2}{\omega_n^2} \right) \]

Assuming the F.C.'s are zero, \( \dddot{x}(0) = 0 \)

\[ \ddot{x}(0) = 0 = A \]
\[ \dddot{x}(0) = 0 = B \omega_n - \frac{a_0}{\omega_n^2} \Rightarrow B = \frac{a_0}{\omega_n^3} \]

Then,

\[ \dddot{x} = \frac{a_0}{\omega_n^3} \sin \omega_n t - \frac{a_0 t}{\omega_n^2} \]

\[ \dddot{x} = \frac{a_0}{\omega_n^2} \left[ \frac{\sin \omega_n t}{\omega_n} - t \right] \]

The relative acceleration \( \dddot{z} \) is:

\[ \dddot{z} = -\frac{a_0}{\omega_n^2} \sin \omega_n t \]

Then, \( \dddot{z} + \frac{k}{m} z = -\frac{a_0}{\omega_n^2} \sin \omega_n t + a_0 t \)

\[ \dddot{x} = a_0 t \left[ 1 - \frac{1}{\omega_n^2} \sin \omega_n t \right] \]

Solution to Part (a)

For \( z \) to be small, \( \omega_n \) should be large. Since \( \omega_n = \sqrt{k/m} \), \( k/m \) should be large for stiff support structure.