Coordinate Coupling (Dynamic and Static)

Non-diagonal mass matrix → system said to be dynamically coupled or have inertia coupling

Non-diagonal stiffness matrix → system said to be statically coupled or have elastic coupling.

Often, the nature of the coupling depends on the the choice of coordinates and how the system is described mathematically.

Example: Determine the equations of motion for the following system.

For small oscillations, \( x = x + \theta \)

System vibration with general plane motion, which is a combination of translation plan rotation.

In matrix form, Eqs. 1 and 2 become:

\[
\begin{bmatrix}
\begin{bmatrix} m_a & m_a \\
\end{bmatrix} \begin{bmatrix} x \\
\end{bmatrix} + \begin{bmatrix} (k_1+k_2) & k_3 \\
\end{bmatrix} \begin{bmatrix} \theta \\
\end{bmatrix} & \begin{bmatrix} \mathbf{0} \\
\end{bmatrix} + \begin{bmatrix} (m_a+m_a) & m_a \times l^2 \\
\end{bmatrix} \begin{bmatrix} \theta \\
\end{bmatrix} = \begin{bmatrix} \mathbf{0} \\
\end{bmatrix}
\end{bmatrix}
\]

E.F.D. of the system:
Coordinate System 2:

\[
\begin{bmatrix}
\mathbf{m} & \mathbf{0} \\
\mathbf{0} & \mathbf{I}
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{\theta}
\end{bmatrix} =
\begin{bmatrix}
(k_x + k_b) & (k_b - k_a) \\
(k_b - k_a) & (k_b + k_a)
\end{bmatrix}
\begin{bmatrix}
x \\
\theta
\end{bmatrix}
\]

In matrix form Eqs. (3) and (4) are:

\[m\ddot{x} + (k_x + k_b)x + (k_b - k_a)\theta = 0 \quad (3)\]

\[I\ddot{\theta} + (k_b + k_a)\dot{\theta} + (k_b - k_a)x = 0 \quad (4)\]

Dynamic coupling has been eliminated. Usually, when the linear coordinate \(x\) is the displacement of the mass center and the displacements of other points on the body are expressed in terms of this coordinate and the angular rotation \(\theta\) of the body, dynamic coupling will be avoided.