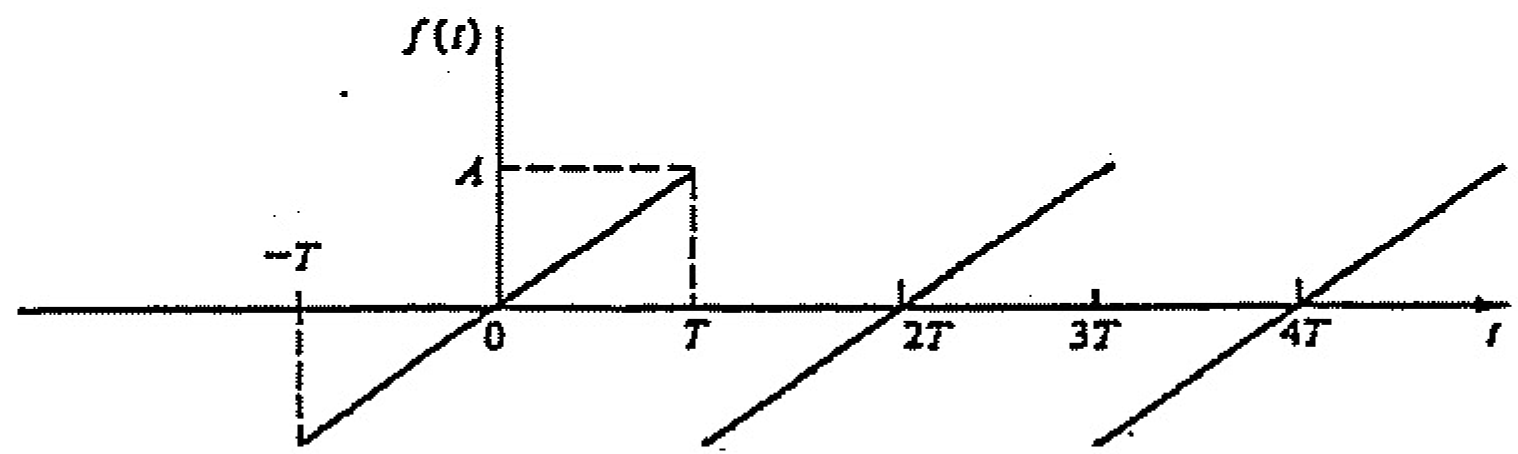
Example 2

Determine the Fourier series for the $f(t)$ shown in the accompanying figure. Sketch the frequency spectrum for $T = 0.05$ s.



$\tau = 2T \qquad \omega = \frac{2\pi}{\tau} = \frac{\pi}{T} \qquad f(t) = At/T$ (odd function)

Since $f(t)$ is odd, $a_n = 0$

$$b_n = \frac{4}{\tau}\int_0^{\tau/2} f(t)\sin\frac{2n\pi t}{\tau}\,dt = \frac{4}{2T}\int_0^{T} A\frac{t}{T}\sin\frac{2n\pi t}{2T}\,dt$$

$$= \frac{2A}{T^2}\int_0^T t\sin\frac{n\pi t}{T}\,dt = \frac{2A}{T^2}\left[\frac{1}{\frac{n^2\pi^2}{T^2}}\sin\frac{n\pi t}{T} - \frac{t}{\frac{n\pi}{T}}\cos\frac{n\pi t}{T}\right]_0^T$$

$$= \frac{2A}{T^2}\left[\cancelto{0}{\frac{T^2}{n^2\pi^2}\sin n\pi} - \frac{T^2}{n\pi}\cos n\pi\right]$$

$$= \frac{2A}{n\pi} \quad (n = 1, 3, 5 \ldots)$$

$$= \frac{-2A}{n\pi} \quad (n = 2, 4, 6, \ldots)$$

$$\boxed{f(t) = \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(-\cos n\pi)\sin\frac{n\pi t}{T}}$$