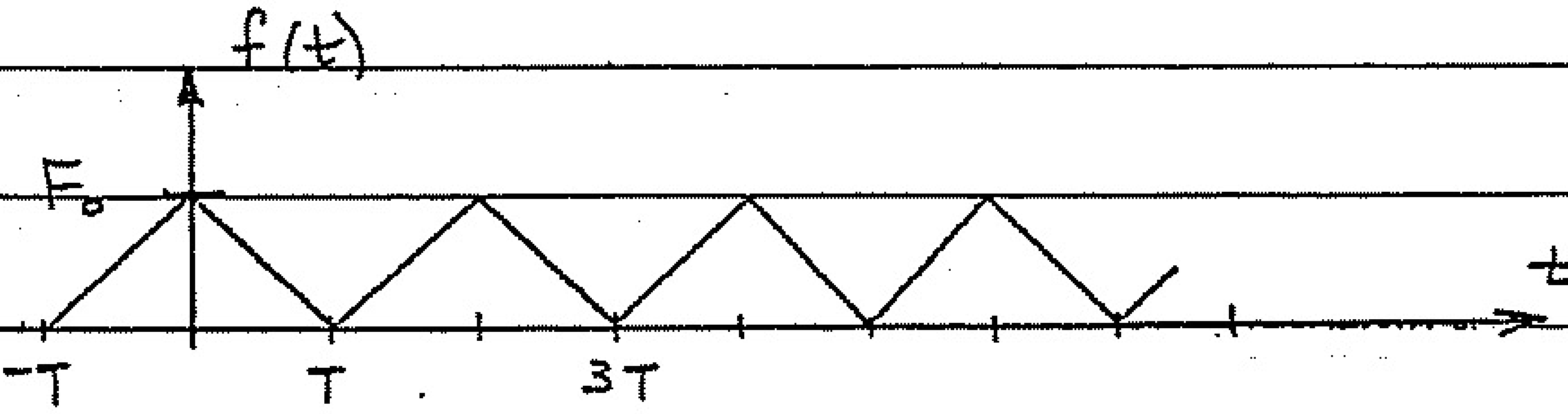
Example 1: Determine the Fourier Series for the triangular wave having the origin shown

$\tau = 2T$

$\omega = \frac{2\pi}{\tau} = \frac{\pi}{T}$

$f(t) = F_o(1 - t/T)$



Since $f(t)$ = even function, $b_n = 0$ and

$$a_n = \frac{4}{\tau}\int_0^{\tau/2} f(t)\cos\frac{2n\pi t}{\tau}\,dt = \frac{4}{2T}\int_0^T F_o(1 - t/T)\cos\frac{2n\pi t}{2T}\,dt$$

$$= \frac{2F_o}{T}\int_0^T (1 - t/T)\cos\frac{n\pi t}{T}\,dt \qquad n = 0, 1, 2, 3 \ldots$$

$$= \frac{2F_o}{T}\left(\frac{T}{n\pi}\right)\sin\frac{n\pi t}{T}\Big|_0^T - \frac{2F_o}{T^2}\frac{T^2}{(n\pi)^2}\left[\cos\frac{n\pi t}{T} + \frac{n\pi t}{T}\sin\frac{n\pi t}{T}\right]\Big|_0^T$$

$$a_n = \frac{4F_o}{(n\pi)^2} \quad (n = 1, 3, 5 \ldots) \qquad n \neq 0$$

$$a_n = 0 \quad (n = 2, 4, 6 \ldots)$$

$$\frac{a_o}{2} = \frac{2}{\tau}\int_0^{\tau/2} f(t)\,dt = \frac{2}{2T}\int_0^T F_o(1 - t/T)\,dt = \frac{F_o}{T}\left[\left(t - \frac{t^2}{2T}\right)\right]_0^T = \frac{F_o}{2}$$

$$\therefore\ f(t) = \frac{F_o}{2} + \frac{4F_o}{\pi^2}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^2}\cos\frac{n\pi t}{T}$$

$$f(t) = F_o\left[\frac{1}{2} + \frac{4}{\pi^2}\left(\cos\frac{\pi t}{T} + \frac{1}{3^2}\cos\frac{3\pi t}{T} + \ldots\right)\right]$$

Note: $F/2$ is the mean value of $f(t)$