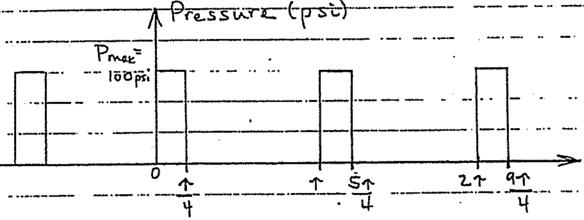
Determine the Fourier expansion for the following function:



$$p(t) = 100 \qquad 0 \le t \le \tau/4$$
$$p(t) = 0 \qquad \tau/4 \le t \le \tau \quad , \qquad \text{Period} = \tau$$

$$\frac{a_0}{2} = \frac{1}{\tau}\int_0^{\tau} p(t)\,dt = \frac{1}{\tau}\int_0^{\tau/4} 100\,dt = \frac{1}{\tau}\,100(t)\Big|_0^{\tau/4} = \frac{100}{\tau}\,\frac{\tau}{4} = 25$$

$$a_n = \frac{2}{\tau}\int_0^{\tau/4} 100\cos\frac{2n\pi t}{\tau}\,dt = \frac{200}{\tau}\,\frac{\sin\frac{2n\pi t}{\tau}}{\frac{2n\pi}{\tau}}\Bigg|_0^{\tau/4}$$

$$= \frac{200}{\tau}\,\frac{\tau}{2n\pi}\left[\sin\frac{2n\pi}{4} - 0\right] = \frac{100}{n\pi}\sin\frac{n\pi}{2}$$

$$= 0 \quad (n = \text{even}) \qquad = \frac{100}{n\pi}\sin\frac{n\pi}{2} \quad (n = \text{odd})$$

$$b_n = \frac{2}{\tau}\int_0^{\tau/4} 100\sin\frac{2n\pi t}{\tau}\,dt = -\frac{200}{\tau}\,\frac{\cos\frac{2n\pi t}{\tau}}{\frac{2n\pi}{\tau}}\Bigg|_0^{\tau/4}$$

$$= \frac{-200}{\tau}\,\frac{\tau}{2n\pi}\left[\cos\frac{2n\pi}{4} - 1\right] = \frac{100}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

Solving for $p(t)$:

$$p(t) = 25 + \sum_{n=1}^{\infty}\left(\frac{100}{n\pi}\right)\left[\sin\frac{n\pi}{2}\cos\frac{2n\pi t}{\tau} + \left(1 - \cos\frac{n\pi}{2}\right)\sin\frac{2n\pi t}{\tau}\right]$$

Consider, $\sin\frac{n\pi}{2} = (-1)^{\frac{(n-1)}{2}}$ $\quad n = \text{odd}$

$\cos\frac{n\pi}{2} = (-1)^{n/2}$ $\quad n = \text{even}$

so,

$$p(t) = 25 + \sum_{n=1,3,5\ldots}^{\infty} \frac{100}{n\pi}(-1)^{\frac{(n-1)}{2}} \cos\frac{2n\pi t}{T}$$

$$+ \sum_{n=2,4,6\ldots}^{\infty} \frac{100}{n\pi}\left[1-(-1)^{n/2}\right]\sin\frac{2n\pi t}{T} + \sum_{n=1,3,5\ldots}^{\infty} \frac{100}{n\pi}\sin\frac{2n\pi t}{T}$$

Alternate Solution: (Complex Notation)

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-j2n\pi t/T}\, dt = \frac{1}{T}\int_0^{T/4} 100\, e^{j2n\pi t/T}\, dt$$

$$= \frac{100\,T}{-jT\,2n\pi}\, e^{-j2n\pi t/T}\Big|_0^{T/4} = \frac{50}{-jn\pi}\left[e^{-j\frac{n\pi}{2}} - 1\right]$$

$$C_n = \frac{50j}{n\pi}\left[e^{-jn\pi/2} - 1\right]$$

$$C_0 = \frac{1}{T}\int_0^{T/4} 100\, dt = 25$$

$$f(t) = 25 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=+\infty} \frac{50j}{n\pi}\left[e^{-jn\pi/2} - 1\right] e^{j2n\pi t/T}$$

$$a_n = C_n + C_{-n}$$

$$= \frac{50j}{n\pi}\left[e^{-jn\pi/2} - 1\right] - \frac{50j}{n\pi}\left[e^{jn\pi/2} - 1\right]$$

$$= \frac{50j}{n\pi}\left[e^{-jn\pi/2} - 1 - e^{jn\pi/2} + 1\right]$$

$$= \frac{50j}{n\pi}\left[e^{-jn\pi/2} - e^{jn\pi/2}\right] = \frac{100}{j\,2n\pi}\left[e^{jn\pi/2} - e^{-jn\pi/2}\right]$$

$$= \frac{100}{n\pi}\sin\frac{n\pi}{2}$$ ✓ checks.

$$b_n = j\left(C_n - C_{-n}\right)$$

$$= \frac{-50}{n\pi}\left[e^{-jn\pi/2} - 1\right] - \left(\frac{50}{n\pi}\right)\left[e^{jn\pi/2} - 1\right]$$

$$= \frac{-50}{n\pi}\left[e^{-jn\pi/2} - 1 + e^{jn\pi/2} - 1\right]$$

$$= \frac{-100}{n\pi}\left[\frac{e^{-jn\pi/2} + e^{jn\pi/2}}{2} - \frac{2}{2}\right]$$

$$= \frac{-100}{n\pi}\left[\cos\frac{n\pi}{2} - 1\right]$$ ✓ checks.