The uniform rod of length $l$ and cross-sectional area $A$ is to be modeled by four lumped masses. The material of the rod has a modulus of elasticity $E$ and a mass density of $\rho$. Any two different models are to be considered for axial vibrations.

**Exact Solution:** From the solution of the wave equation one obtains:

$$
\omega_n = \frac{n\pi}{l} \sqrt{\frac{E}{\rho}} \quad (\text{rad/s}) \quad n = 1, 2, 3, \ldots
$$

(Undamped natural frequencies)

**Lumped-Mass Solution:**

The stiffness matrix $[k]$ is the same for both models.

$$
k = \frac{AE}{l_4} = \frac{4AE}{l} \quad (\text{each} \: k_i)
$$

---

### Model 1:

$$
m_1 = m_2 = m_3 = m_4 = \frac{AE}{4}
$$

### Model 2:

$$
m_1 = \frac{AE}{4}
$$
The mass matrices for the two models are:

Model 1:

\[
\begin{bmatrix}
0.25 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0 \\
0 & 0 & 0.25 & 0 \\
0 & 0 & 0 & 0.25
\end{bmatrix}
\]

Model 2:

\[
\begin{bmatrix}
0.25 & 0 & 0 & 0 \\
0 & 0.25 & 0 & 0 \\
0 & 0 & 0.25 & 0 \\
0 & 0 & 0 & 0.125
\end{bmatrix}
\]

Note: For a 5 lumped-mass model:

\[
\alpha = \frac{1}{\sqrt{\frac{E}{A}}}
\]

Summary: Model 2 is the best.