Formulation of the Equations of Motion:

We will be using these basic methods:

I.) Newton's Second Law:
\[ F = m \frac{d^2x}{dt^2} \]
\[ F = m \frac{d^2r}{dt^2} \]
\[ \Sigma F = m \frac{d^2x}{dt^2} \]
\[ \Sigma F = m \frac{d^2r}{dt^2} \] etc.

II.) Work and Energy:

Conservation of Energy says:
\[ T + V = T_0 + V_0 \]
\[ T + V = \text{constant} \]

For a conservative system:

If you have a non-conservative system:
\[ T_1 + V = T - T_2 \]

III.) Lagrange's Equation and Hamilton's Principle:

Lagrange's:
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \ddot{q}_i \]
\[ i = 1, 2, \ldots, n \]

Hamilton's:
\[ \sum T (\dot{q}) dt + \sum W_{nc} dt = 0 \]

Where:
- \( T \) = variation taken during indicated time interval
- \( W_{nc} \) = work done by non-conservative forces.

Example:

Radius = r
Mass = m

Determine the differential equation of motion of the solid cylinder. Assume no slipping.

(A sphere rolling with slipping is a special case of general plane motion. An can consider the point of contact as a fixed point.)

\[ F = m \omega \]
\[ F = c \ddot{\theta} \]

\[ \Sigma H = I \alpha \]

\[ \dot{\alpha} \]

\[ I \ddot{\theta} + c \dot{\theta} + k \dot{\theta} = 0 \]

\[ \frac{I \ddot{\theta} + c \dot{\theta} + k \dot{\theta}}{I} = 0 \]

\[ T_0 \dot{\theta} + c \dot{\theta} + k \dot{\theta} = 0 \]
The relationship \( x = r \theta \) and its derivatives can be substituted in, yielding:

\[
\ddot{x} + \frac{c}{l_0} \dot{x} + \frac{k}{m_0} x = 0
\]

Each of the differential equations could have been derived by summing forces about the mass center and summing forces in the \( x \)-direction, then summarizing the two equations.

Note: The above equations could have been derived using:

\[
\begin{align*}
F_x &= k_x x \\
F_y &= k_r \theta
\end{align*}
\]

General form of the differential equation given by:

\[
\ddot{x} + 2 \zeta \omega_n \dot{x} + \omega_n^2 x = 0
\]

Case 1: Undamped

\[
x + \frac{k}{m} x = 0 \quad \text{or} \quad \ddot{x} + \omega_n^2 x = 0
\]

Solution:

\[
x = A \cos \omega_n t + B \sin \omega_n t
\]

Initial Conditions:

\[
\begin{align*}
x(0) &= x_0 \\
\dot{x}(0) &= \dot{x}_0
\end{align*}
\]

- Use initial conditions to determine \( A + B \)
- Also can add harmonic components of solution to obtain single harmonic response with phase angle

\[
x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t \quad \text{or} \quad x = x_0 \cos (\omega_n t - \theta)
\]

\[
\theta = \tan^{-1} \frac{\dot{x}_0}{x_0}
\]