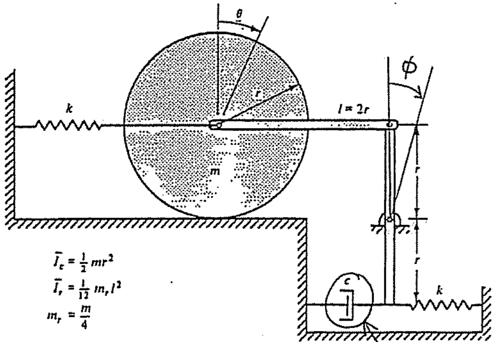
Example 3

Use the energy method, $T_{max} = U_{max}$, to determine the undamped natural frequency ω_n for the system.



$r\theta = r\phi$

$\theta = \phi$

$\theta = \theta_{max} \sin \omega_n t$

$\dot{\theta} = \theta_{max} \omega_n \cos \omega_n t$

$$U_{max} = \frac{1}{2} k (r\theta_{max})^2 + \frac{1}{2} k (r\phi_{max})^2$$

$$U_{max} = k(r\theta_{max})^2 = kr^2 \theta_{max}^2$$

$$T_{max} = \frac{1}{2} \bar{I}_c (\underbrace{\theta_{max}\omega_n}_{\dot{\theta}_{max}})^2 + \frac{1}{2} m (\underbrace{r\theta_{max}\omega_n}_{(v_c)_{max} = r\dot{\theta}_{max}})^2 \qquad \dot{\phi}_{max} = \dot{\theta}_{max}$$

$$+ \frac{1}{2} m_r (\underbrace{r\theta_{max}\omega_n}_{(v_c)_{max}})^2 + \frac{1}{2} \bar{I}_r (\underbrace{\phi_{max}\omega_n}_{})^2$$

$$T_{max} = \frac{1}{2} \theta_{max}^2 \omega_n^2 \left[\frac{mr^2}{2} + mr^2 + \frac{m}{4} r^2 + \frac{1}{12} mr^2 \right] = \theta_{max}^2 \omega_n^2 \left(\frac{11 mr^2}{12} \right)$$

Setting $T_{max} = U_{max}$

$$\theta_{max}^2 \omega_n^2 \left(\frac{11 mr^2}{12} \right) = kr^2 \theta_{max}^2$$

$$\boxed{\omega_n = \sqrt{\frac{12k}{11m}}}$$