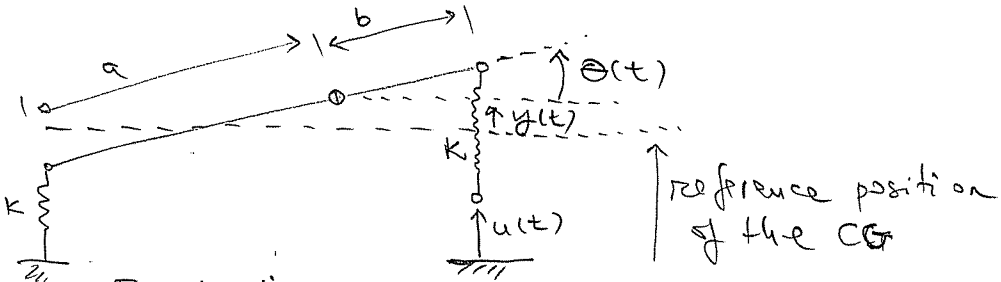
Highway Markers Example.



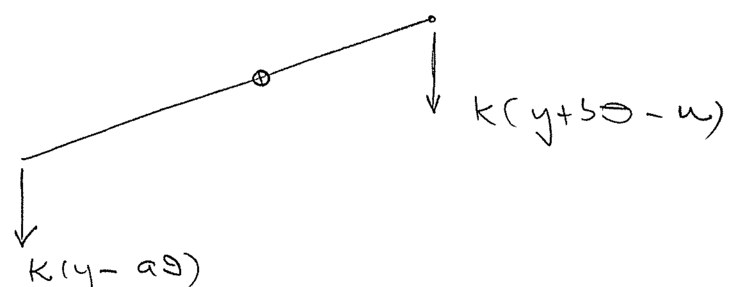
Front view

Small angle assumption holds

The vehicle moves up from the reference position by $y(t)$. It also rolls by $\theta(t)$.

We have two degrees of freedom: y & θ.

EOM: based on Newton's second law



Translation:

$$m\ddot{y} = -k(y-a\theta) - k(y+b\theta-u)$$

Rotation:

$$J\ddot{\theta} = [k(y-a\theta)]a - [k(y+b\theta-u)]b$$

Then,

$$\begin{cases} m\ddot{y} + 2ky + (-ka+kb)\theta = ku \\ J\ddot{\theta} + (-ka+kb)y + (ka^2+kb^2)\theta = kub \end{cases}$$

We know that:

$m = 1000$ $\qquad k_1 = k_2 = k = 5\cdot10^4$

$J = 200$ $\qquad a = 1.75 \quad b = 1.25$

In matrix form:

$$\begin{bmatrix} 1000 & 0 \\ 0 & 200 \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 10^5 & -2.5\cdot10^4 \\ -2.5\cdot10^4 & 23.125\cdot10^4 \end{bmatrix} \begin{Bmatrix} y \\ \theta \end{Bmatrix} = \begin{Bmatrix} 10^4 u(t) \\ 1.25\cdot10^4 u(t) \end{Bmatrix}$$

consequently,

$m_1 = 1000 \quad m_2 = 200 \quad k_{11} = 10^5 \quad k_{12} = -2.5 \cdot 10^9 \quad k_{22} = 23.125 \cdot 10^4$

computing the natural frequencies:

$$1000 \cdot 200 \cdot \omega^4 - (1000 \cdot 23.125 \cdot 10^4 + 200 \cdot 10^5) + 23.125 \cdot 10^9 - 6.25 \cdot 10^8 = 0$$

Solving this equation leads to:

$\omega_{n(1)} = 9.8514 \quad$ rad/sec

$\omega_{n(2)} = 34.047 \quad$ rad/sec.

First vibration ratio:

$$r_{(1)} = -\frac{k_{12}}{k_{22} - m_2 \omega_{n(1)}^2} = \frac{2.5 \cdot 10^9}{23.125 \cdot 10^4 - 200 \cdot (9.8196)^2} = 0.1180$$

Likewise,

$r_{(2)} = -42.368$

Modal vectors:

$$\{u\}_{(1)} = \begin{Bmatrix} 1 \\ 0.1180 \end{Bmatrix} \qquad \{u\}_{(2)} = \begin{Bmatrix} 1 \\ -42.368 \end{Bmatrix}$$

Distance between workers:

65 mph, speed

$$\tau_{(1)} = \frac{2\pi}{\omega_{n(1)}} = \frac{L_{(1)}}{V} \Rightarrow L_{(1)} = \frac{2\pi \cdot V}{\omega_{n(1)}} = \frac{2\pi \cdot 29.0578}{9.8514} = 18.5329 \text{ m}$$

$$L_{(2)} = \frac{2\pi \cdot 29.0579}{34.047} = 5.3624 \text{ m.}$$